

TOMISLAV PROKOPEC ^{†‡}

*Lancaster University, Physics Department
Lancaster, LA1 4YB, England, UK*

ABSTRACT

In this lecture first I present a brief genesis of the ideas on the electroweak baryogenesis and then I focus on a mechanism in which the source of CP violation is a CP -violating field condensate which could occur, for example, in multi-higgs extensions of the Standard Model. In the limit of a thick bubble wall one finds a classical force acting on particles proportional to the mass squared and the CP violating phase. One can study this force in the fluid approximation in which the effects of transport and particle decays can be taken into account. A novelty in this talk is generalization of the problem to the relativistic velocity. There is a regime in which the final formula for the baryon asymmetry has a rather simple form.

1. Prelude ..

In this lecture I will present some of the recent developments in understanding baryogenesis at the electroweak phase transition.

I will focus on the work I have done in collaboration with Michael Joyce and Neil Turok. The main idea is rather simple:

In the semiclassical approximation baryogenesis can be described by a CP violating classical field condensate. The effect of this condensate can be studied in the fluid approximation which takes account of both transport and particle decays.

I will also present some of the hystorical ideas that ‘seeded’ our work as well some of the recent developments that I find related or interesting. (I cannot hope for a lack of personal bias in my choice.)

2. Sakharov’s Conditions and the Electroweak Theory

The baryon-to-entropy ratio of the Universe one would like to explain is

$$\frac{n_B}{s} = (4 - 7)10^{-11} \quad (1)$$

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[†]Based on collaboration with Michael Joyce and Neil Turok

[‡]e-mail: T.Prokopec@lancaster.ac.uk

This number is a consistency constraint for nucleosynthesis calculations be in agreement with the observed primordial elements' abundances. For a pedagogical review see ², for recent developments ³.

'Before Sakharov' baryogenesis was a fine tuning problem of the initial conditions of the Universe. Sakharov realized ¹ that baryons can be generated *dynamically* provided the following conditions are satisfied

- ▷ *Condition 1*: there exist processes that violate baryon number
- ▷ *Condition 2*: system is out of thermal equilibrium
- ▷ *Condition 3*: there exist both *C* and *CP* violating processes

The necessity of *Condition 1* is obvious.

An elegant proof of *Condition 2* can be found in ⁴. In thermal equilibrium the hamiltonian \mathcal{H} is *CPT* invariant, and the baryon number B is odd under *CPT*

$$(CPT) \mathcal{H} (CPT)^{-1} = \mathcal{H}, \quad (CPT) B (CPT)^{-1} = -B \quad (2)$$

In thermal bath baryon number is given by a thermal average

$$\langle B \rangle = \text{Tr} [e^{-\beta \mathcal{H}} B] \quad (3)$$

We can now insert $(CPT)(CPT)^{-1} = 1$ into the trace and use Eq. (2) to get

$$\begin{aligned} \langle B \rangle &= \text{Tr} [(CPT) e^{-\beta \mathcal{H}} B (CPT)^{-1}] \\ &= \text{Tr} [e^{-\beta \mathcal{H}} (CPT) B (CPT)^{-1}] = -\langle B \rangle \end{aligned} \quad (4)$$

so $\langle B \rangle = 0$.

Next I present an argument which indicates the necessity of *CP* violation, as stated by *Condition 3*. (A similar proof as above applies, but I want to avoid the use of thermal equilibrium.) Recall since the net baryon number is defined as the difference between the number of baryons and antibaryons $B = b - \bar{b}$ and under *CP*: $b \rightarrow \bar{b}$ and $\bar{b} \rightarrow b$, we have

$$(CP) B (CP)^{-1} = -B \quad (5)$$

In case there is no *CP* violation the processes which create net b occur with the same rate as the processes that create net \bar{b} and *no* net B will be created.

Are the Sakharov conditions realized in the Standard Model?

We start with the baryon number violation. The Weinberg-Salam theory has on the quantum level a remarkable axial current anomaly through which baryon number current is violated

$$\begin{aligned} \partial_\mu J_B^\mu &= N_F \frac{g_w^2}{32\pi^2} [-W_{\mu\nu}^a \tilde{W}^{a\mu\nu}] + N_F \frac{g_1^2}{32\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu} \\ &\equiv N_F \frac{g_w^2}{32\pi^2} [-\partial_\mu K^\mu + \partial_\mu k^\mu] \\ K^\mu &= \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left[W_\nu \partial_\rho W_\sigma + \frac{2}{3} i g_w W_\nu W_\rho W_\sigma \right] \\ k^\mu &= \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} [B_\nu \partial_\rho B_\sigma] \end{aligned} \quad (6)$$

where $N_F = 3$ is the number of families, $W_{\mu\nu}$ and $B_{\mu\nu}$ and W_μ and B_μ denote the $SU(2)_L$ and $U(1)_Y$ field strengths and fields, respectively, and $\tilde{W}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}W^{\rho\sigma}$ denotes the dual field strength. The fact that $\partial_\mu K^\mu$ and $\partial_\mu k^\mu$ are total derivatives does not mean that this anomalous current violation can be gauged away without any physical consequences by merely shifting the current. The resolution is in the nontrivial topological structure of the theory. The integral of Eq. (6) defines how the topology changes in the gauge fields related to the changes of the baryon number:

$$\Delta B = N_F \Delta n_{CS}, \quad n_{CS} = -\frac{g_w^2}{32\pi^2} \int K^\mu dS_\mu \quad (7)$$

At zero temperature quantum tunnelling can generate baryons ⁵ but the rate is tiny $\sim e^{-4\pi/\alpha_w}$ – it would not create even one baryon in all of the (visible) Universe!

The discovery of the sphaleron ⁶, a classical solution to the dimensionally reduced action which carries a half topological charge and as such, it was argued, may mediate baryon number violating processes, lead to a belief that the finite temperature baryon number violating rate may be unsuppressed ⁷ which would make the electroweak baryogenesis possible. This was in fact proven by Arnold and McLerran in ⁸ where they showed that in the unbroken phase the number of sphaleron transitions (per unit volume and time) reads

$$\frac{\Gamma_{sph}}{V} \sim \frac{E_{sph}^7}{T^3} e^{-\frac{E_{sph}}{T}}, \quad E_{sph} = A(\lambda/g_w^2) \frac{2m_w(T)}{\alpha_w} \quad (8)$$

where $A(\lambda/g_w^2) \sim 1.5 - 2.7$; in fact for physically interesting range of λ/g_w^2 to a good approximation $A \sim 1.7$. I have not bothered to quote all pre-exponential factors in this relationship; they can be found in ⁸. The main point is the sphaleron rate is exponentially suppressed by the Boltzmann factor with the sphaleron free energy in the exponent, which is defined as the saddle point of the dimensionally reduced three dimensional action.

An important condition which ought to be satisfied in order for any electroweak baryogenesis model be viable is

•► *Sphaleron erasure*: The baryons generated at the transition must not be washed out by the subsequent sphaleron transitions in the broken phase ⁹.

Roughly speaking this means that the sphaleron rate per unit time Γ_{sph} in the broken phase ought to be smaller than the expansion rate of the Universe H . More precisely, since at the time of the phase transition completion, the higgs expectation value $\phi_0 = \langle \phi \rangle$ still grows quite rapidly on the expansion time scale, sphalerons are active only a small portion of the expansion time after the completion ¹⁰. With this in mind one gets slightly less stringent bound on the sphaleron energy

$$E_{sph} \geq 35T \quad (9)$$

Taking account of $M_w = g_w \phi_0/2$ and the dependence of the higgs expectation value on the couplings (which is roughly $\phi_0 \sim g^3 T/\lambda_T$) one gets an upper bound on the

higgs mass. It turns out that with the two loop effective potential this leads to a bound: $m_H < 30\text{GeV}$. If one allows for nonperturbative effects this bound becomes less stringent. Recall that the current constraint on the higgs mass is $m_H > 60\text{GeV}$ for the Standard Model higgs. Fortunately this does not mean the end of electroweak baryogenesis. Since we are interested in an extended higgs sector which contains CP violation and the constraints on the mass of the lightest higgs are much less stringent, and the above calculation should be altered accordingly, I believe that at the moment no serious discrepancy exists.

An important calculation of the ‘sphaleron’ rate in the unbroken phase is still missing due to the ill understanding of the infrared sector of gauge theories. Indeed it is believed that magnetic fields, which are involved in the processes, are screened at the scale of the infamous magnetic mass: $m_{mag} \sim g_w^2 T$, which specifies the ‘magnetic’ field correlation length $\xi \sim 1/g_w^2 T$.

We now present a simple estimate of the rate. There will be instantons of all sizes $\rho \in \{0, \xi\}$ which represent tunneling trajectories in the configuration space with the barrier height of order $1/\alpha_w \rho$. The smallest barrier is for large instantons $\rho \sim \xi$ with the energy $\sim T$ so that there is no exponential suppression to tunnelling; the rate is then determined by the prefactor which is of order ξ^4 , hence $\Gamma_{sph}/V \sim (\alpha_w T)^4$.

A number of numerical studies ¹¹ supports this naive argument

$$\frac{\Gamma_{sph}}{V} = \kappa_{sph} (\alpha_w T)^4, \quad \kappa_{sph} \sim 0.1 - 1 \quad (10)$$

There are however problems with the simulation. Ambjorn *et al.* use the real time classical theory which is plagued with ultraviolet divergences; the authors hope that the lattice *cut-off* takes care of that. Also the Gauss constraint is not naturally imposed. I will take Eq. (10) as a guidance to the true rate.

Since the symmetric phase ‘sphaleron’ rate Eq. (10) is much faster than the broken phase rate Eq. (8) an efficient baryogenesis mechanism should generate baryons in the symmetric phase. This is indeed possible if one takes account of particle transport.

Regarding *Condition 2* perturbative calculations ¹² valid for a rather light higgs particle $m_H \leq 70\text{GeV}$ indicate that the phase transition is first order and likely proceed *via* bubble nucleation providing the required departure from thermal equilibrium. To resolve the problem fully one requires to solve for nonperturbative effects which is possible only by using lattice simulation. Recently ¹³ it has been shown that the dimensionally reduced theory is well suited for studying equilibrium properties of the transition in a lattice simulation. A preliminary result ¹⁴ indicates that the phase transition becomes second order for the higgs mass above about 80GeV.

What about *Condition 3*? C is maximally violated. For example no right handed neutrino has been observed.

The main problem for the Standard Model baryogenesis is a small CP violation. A natural measure is the CP violating phase from the Kobayashi-Maskawa matrix $\delta_{CP} \sim 10^{-19}$, and since it is usually argued that $n_B \propto \delta_{CP}$ it seems without an additional CP violation the Standard Model baryogenesis is out! This however may not be true. Recently a couple of attempts have been made to resolve this formidable

problem. Shaposhnikov and Farrar ¹⁵ (inspired by a modest CP violation of the neutral kaon) have realized that under certain conditions the thermal reflection off the bubble wall may lead to a net baryonic current due to the difference in the tree level light quarks' masses. This model, since in its original formulation takes no account of the possible loss of quantum coherence in the reflection off the wall, has been recently jeopardized ^{16, 17}. Another notable attempt to solve the problem of a small CP violation is by Nasser and Turok ¹⁸. Based on a linear response analysis they argue that there might be instability to formation of a Z field condensate on the bubble wall caused by the chiral charge deposit from the reflected top quarks. This condensate as we will see below may generate a net baryon number. The longitudinal Z field domains of the opposite sign form indiscriminately on the wall and no net baryon number is formed unless there is a nonzero CP violation. This gives an advantage to the formation of a particular sign condensate. Diffusion of the one sign domains against the other enhances the bare CP violation. If this mechanism works (as it may for a very heavy top quark) it would result in $n_B \propto \sqrt{\delta_{CP}}$.

How can one bias baryon number production?

If there is for example a term in the free energy (possibly dynamically generated) that couples to the baryon current

$$\Delta F = a_\mu J_B^\mu \quad (11)$$

then a classical formula of the near equilibrium statistical physics gives

$$\dot{n}_B = -\Gamma_{sph}\mu_B, \quad \mu_B = \frac{\delta F}{\delta n_B} = a_0 \quad (12)$$

where μ_B is the chemical potential for the baryon number. (In this light *Condition 2* of Sakharov asserts that in thermal equilibrium $\mu_B = 0$.) Indeed baryon number production is biased if on average $a_0 \neq 0$. This reasoning sets the stage for the next section.

3. The ideas of Cohen, Kaplan and Nelson

There are two ideas of relevance for my work:

- ▷ *spontaneous* baryogenesis and
- ▷ *charge transport*

The idea of the *spontaneous* baryogenesis in its initial form ¹⁹ is rather simple. Cohen and Kaplan assumed that at some early stage in the Universe CPT symmetry of the hamiltonian was temporarily violated by a term of form:

$$\frac{1}{M}\partial_\mu\theta J_B^\mu \rightarrow \frac{1}{M}\dot{\theta}(n_b - n_{\bar{b}}), \quad M \geq 10^{13}\text{GeV}, \quad T < M \quad (13)$$

The implication (\rightarrow) applies for a homogeneous field θ . The *ilion* field $\dot{\theta}$ (the name is supposed to bear reminiscence of the axion) acts as a chemical potential for baryon

number which, when the expansion of the Universe is taken into account, drives $B \neq 0$, hence the name *spontaneous*.

The idea got a new impetus when Dine *et al.* ²⁰ realized that a similar term arises on one-loop level in theories which contain a singlet field (*e.g.* supersymmetric theories). The effective action then acquires a term

$$\frac{1}{3M_{susy}} \partial_\mu s J_B^\mu \quad (14)$$

which may lead to a *spontaneous* baryogenesis at a first order electroweak phase transition. Note that in this scenario the electroweak transition is used to drive the system out of equilibrium. A similar term occurs at one loop in two higgs doublet theories with CP violation in the higgs sector ²¹. But then Cohen, Kaplan and Nelson ²² realized that in these theories even at the *tree* level there is a term

$$y_F \partial_\mu \theta J_F^\mu, \quad J_F^\mu = \bar{\Psi} \gamma^\mu \gamma_5 \Psi \quad (15)$$

which couples the fermionic hypercharge current J_F^μ , which is closely related to the baryonic current, with a CP violating field θ , the gauge invariant relative higgs phase. θ acts as a chemical potential for the baryon number. On an expanding bubble wall θ is predominantly of one sign, as given by the *classical* equations of motion, and hence *it* drives the sphaleron rate in a definite direction creating baryons. As originally designed this mechanism is not manifestly mass suppressed, *i.e.* baryon number does not vanish in the limit when fermion masses go to *zero*. In this limit the chiral symmetry is restored and baryon creation *must* stop. I will present a resolution ²³ of this paradox below.

The idea of *charge transport* was designed ²⁴ in a model with heavy (majorana) neutrinos and explicit CP violation in the coupling constants. In this model CP violation takes place on the bubble wall. As a result of a CP violating reflection on the wall an axial current is injected and *transported* into the symmetric phase where the sphaleron rate is unsuppressed Eq. (10). This mechanism takes advantage of a large CP violation on the bubble wall and of a fast baryon number violating rate in the symmetric phase so it is potentially very efficient. Indeed in the subsequent work Cohen, Kaplan and Nelson ²⁵ have applied the idea to the top quark and managed to generate baryon asymmetry as large as $\sim 10^{-6} \theta_{CP}$. Since the injected reflected chiral top current is the result of a coherent quantum reflection, it is essential that top quarks do not scatter on the bubble wall. Also a significant reflection occurs only when the wall is thinner than m_t^{-1} . Both of these lead to a rather stringent constraint on the wall thickness $L_w \leq 2/T$ which would require a very strong first order phase transition and this is, according to perturbative calculations of the effective thermal higgs potential ¹², highly unrealistic.

We have realized ²⁶ that this idea might have a ‘brighter future’ if applied to the particles that couple weakly to the wall and scatter infrequently. Our favorite candidate is the right-handed τ -lepton since it has a long scattering length and diffuses

very efficiently. We have shown that with $\theta_{CP} \sim 1$ the baryon number produced may be consistent with Eq. (1).

4. Two higgs doublet model and Z condensate

In order to study the effects of CP violating condensate on the particles' dynamics on the wall I will rewrite the Lagrangian in a convenient form so that the interesting CP violation is expressed in the form of a CP violating Z field condensate.

Consider first the higgs kinetic term. I can pick a gauge in which the background classical solution reads

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi_1 e^{i\theta_1} \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi_2 e^{i\theta_2} \end{pmatrix} \quad (16)$$

In what follows I will ignore the electromagnetic fields ($U(1)_{EM}$ remains unbroken so there is no obvious reason for a condensate to appear), the charged W^\pm gauge bosons and the higgs excitations both neutral and charged. Then the higgs kinetic term can be diagonalized

$$\begin{aligned} |\mathcal{D}\Phi_1|^2 + |\mathcal{D}\Phi_2|^2 &= \frac{1}{2} \frac{\phi_1^2 \phi_2^2}{\phi^2} [\partial_\mu (\theta_1 - \theta_2)]^2 + \frac{\phi^2}{2} \left[\frac{g}{2} Z_\mu - \frac{1}{\phi^2} (\phi_1^2 \partial_\mu \theta_1 + \phi_2^2 \partial_\mu \theta_2) \right]^2 \\ &\equiv \frac{\phi_1^2 \phi_2^2}{2\phi^2} \partial_\mu \theta \partial^\mu \theta + \frac{\phi^2}{2} \left(\frac{g}{2} Z_{GI} \right)^2 \\ gZ_\mu &= g_2 W_\mu^3 - g_1 B_\mu, \quad \phi^2 = \phi_1^2 + \phi_2^2, \quad g^2 = g_1^2 + g_2^2 \end{aligned} \quad (17)$$

where I ignored the kinetic terms for ϕ_1 and ϕ_2 . Note that when either of the higgs *vevs* vanish the gauge invariant relative phase θ loses its meaning, as it should. Both θ and Z_μ^{GI} are gauge invariant, and we take them as our definition of the CP violating condensate fields.

Consider now the fermionic part of the Lagrangian. Assume for simplicity that only Φ_1 couples to the fermions *via* the Yukawa terms. The phase θ_1 can be removed by performing an anomaly free rotation proportional to $[T^3 - Y + \frac{1}{2}(B - L)]$ on the fermions (and also on the Higgs field) at the cost of introducing a new term in the fermionic kinetic term

$$\bar{\Psi} i \gamma^\mu (\partial_\mu - i g_A \tilde{Z}_\mu \gamma_5) \Psi - m \bar{\Psi} \Psi \quad (18)$$

where $g_A = +g/4$ ($g^2 = g_1^2 + g_2^2$) for up-type quarks and (left-handed) neutrinos, and $g_A = -g/4$ for down-type quarks and charged leptons, and

$$g_A \tilde{Z}_\mu = g_A Z_\mu^{GI} - \frac{\phi_2^2}{2\phi^2} \partial_\mu \theta \quad (19)$$

The advantage of this form is that I have reduced the effect the higgs phase to a pure gauge Z field condensate in the presence of a (real) mass $m = y\phi_1/\sqrt{2}$. Since the

axial symmetry is broken this field produces an interesting physical effect which we want to study. This formulation is suitable for studying the effect of a CP violating condensate in the semiclassical limit.

An alternative formulation is to study the classical evolution of the CP violating phase accross the wall which actually neglects the plasma effects. The two-higgs-doublet higgs potential may contain CP violating terms

$$\lambda_5 \left(\mathcal{R} [\Phi_1^\dagger \Phi_2] - v_1 v_2 \cos \xi_0 \right)^2 + \lambda_6 \left(\mathcal{I} \Uparrow [\Phi_1^\dagger \Phi_2] - v_1 v_2 \sin \xi_0 \right)^2 \quad (20)$$

so that the classical evolution of the relative higgs phase

$$\partial_\mu \partial^\mu \theta - \phi^2 \frac{v_1 v_2}{\phi_1 \phi_2} \left[\lambda_5 \cos \xi_0 \sin \theta + \lambda_6 \sin \xi_0 \cos \theta + \frac{\phi_1 \phi_2}{v_1 v_2} (\lambda_6 - \lambda_5) \sin \theta \cos \theta \right] = 0 \quad (21)$$

For a slow moving wall $\partial_\mu \partial^\mu \theta \sim 0$ can be neglected and we have for the symmetric and broken phases

$$\begin{aligned} \tan \theta &= -\frac{\lambda_6}{\lambda_5} \tan \xi_0, & \phi_1 &= \phi_2 = 0 \\ \cos \xi_0 \tan \theta + \frac{\lambda_6}{\lambda_5} \sin \xi_0 + \left(\frac{\lambda_6}{\lambda_5} - 1 \right) &= 0, & \phi_1 &= v_1, \quad \phi_2 = v_2 \end{aligned} \quad (22)$$

so that only if $\lambda_6 \neq \lambda_5$ there is a net change in θ accross the wall, *i.e.* CP violation and Z condensate. For a small CP violation

$$\Delta \theta \simeq -\frac{(\lambda_6 - \lambda_5) \lambda_6 \cos \xi_0}{\lambda_6^2 \cos^2 \xi_0 + \lambda_5^2 \sin^2 \xi_0} \quad (23)$$

which is an upper limit to change in θ given by Eq. (21). How this may drive baryogenesis was first studied in ²¹ and then in ²².

5. A free particle on the bubble wall

We are now ready to study the motion of a free particle on a bubble wall with a Z field condensate, *i.e.* CP violation. Considering a planar wall stationary so that $\tilde{Z}^\mu = (Z^0, 0, 0, Z_3(z))$.

First using Eq. (18) we write the Dirac equation (in the Fourier space)

$$\left[\not{P} - g_A \tilde{\not{Z}} \gamma_5 - m \right] \Psi = 0 \quad (24)$$

which in the chiral representation (where $\Psi \sim [\Psi_R, \Psi_L]$) can be broken into two equations for the two component right-handed Ψ_R and left-handed Ψ_L spinors

$$\begin{aligned} \left[E - g_A Z^0 - \vec{\sigma} \cdot (\vec{P} - g_A \vec{Z}) \right] \Psi_R + m \Psi_L &= 0 \\ m \Psi_R + \left[E + g_A Z^0 + \vec{\sigma} \cdot (\vec{P} + g_A \vec{Z}) \right] \Psi_L &= 0 \end{aligned} \quad (25)$$

where we used $\gamma_5 \Psi_{R,L} = \pm \Psi_{R,L}$. Setting the determinant to zero leads to the dispersion relation which is simple in the wall frame where $\tilde{Z}^\mu = (0, 0, 0, Z(z))$

$$E = \left[p_\perp^2 + \left(\sqrt{p_z^2 + m^2} \mp g_A Z \right)^2 \right]^{1/2} \quad S^z = \pm \frac{1}{2} \quad (26)$$

S_z is the component of the spin in the z direction, measured in the frame in which p_\perp vanishes. In the WKB approximation, this dispersion relation accurately describes particles as they move across a bubble wall - the local eigenstates in Eq. (26) shall form the basis of our treatment. The particles we are most interested in for baryogenesis are left handed particles (*e.g.* t_L 's), and their antiparticles (\bar{t}_L 's, which are right-handed), because these couple to the chiral anomaly. For large $|p_z|$, these are easily identifiable in terms of the eigenstates in Eq. (26). Note that they couple *oppositely* to the Z field.

The group velocity of a WKB wavepacket is determined from the dispersion relation by $v_z = \dot{z} = \partial E / \partial p_z$, and energy conservation $\dot{E} = 0$ implies $\dot{p}_z = -\partial_z E$. From these it is straightforward to calculate the acceleration

$$\frac{dv_z}{dt} = -\frac{1}{2} \frac{(m^2)'}{E^2} \pm \frac{(g_A Z m^2)'}{E^2 \sqrt{E^2 - p_\perp^2}} + o(Z) \quad (27)$$

where E and p_\perp are constants of motion.

This chiral force provided by the Z field effectively produces a potential well which draws an excess chiral charge onto the wall, and leads to a compensating deficit in a 'diffusion tail' in front of the wall. There is net baryon production because B violation is suppressed on the bubble wall.

6. Fluid equations

I now seek to describe the particle excitations with dispersion relations Eq. (26) as classical fluids. I focus on particles with large $|p_z| \sim T \gg m$ for the following reasons: they dominate phase space, the *WKB* approximation is valid, and the dispersion relation simplifies so one can identify approximate chiral eigenstates. The $S_z = +\frac{1}{2}$, $p_z < 0$ branch, and the $S_z = -\frac{1}{2}$, $p_z > 0$ branch constitute one, approximately left handed fluid L , and the other two branches an approximately right-handed fluid R .

The Boltzmann equation is:

$$d_t f \equiv \partial_t f + \dot{z} \partial_z f + \dot{p}_z \partial_{p_z} f = -C(f) \quad (28)$$

where $\dot{z} = \partial_{p_z} E$ and $\dot{p}_z = -\partial_z E$ are calculated from the Hamilton equations and $C(f)$ is the collision integral.

In order to study a system close to equilibrium, fluid approximation is reasonably accurate, provided

- > the scatterings within a fluid are more frequent then between any two fluids,

•> the distortions in the momentum space are small so that the momentum expansion is well behaved (see below); this is the case *e.g.* for a thick bubble wall.

In the plasma frame the fluid *Ansatz* for the distribution function is

$$f = \frac{1}{e^{E+\delta} \pm 1}, \quad \delta = -\mu + p^\nu \delta_\nu + \dots, \quad \delta^\nu = (-\delta T, 0, 0, v) \quad (29)$$

where I have for convenience set $T = 1$ and truncated the expansion at the vector term. Since the dispersion relationship Eq. (26) is obtained in the wall frame we transform energy and momentum

$$E \rightarrow \gamma_w(E + v_w p_z), \quad p_z \rightarrow \gamma_w(p_z + v_w E), \quad p_\perp \rightarrow p_\perp \quad (30)$$

which gives Eq. (29) in the wall frame

$$f = \frac{1}{e^{\gamma_w(E+v_w p_z)+\delta} \pm 1}, \quad \delta = -\mu - \gamma_w(E + v_w p_z)\delta T - \gamma_w(p_z + v_w E)v \quad (31)$$

There is a subtlety in deriving the fluid equations. In the zero mass limit there must be no observable effect (particle number perturbation) due to a nonzero Z field since it is pure gauge. However $d_t p_z = -\partial_z E = -(p_z \pm g_A Z)/E \partial_z(g_A Z)$ would generate a chemical potential. This is what Cohen, Kaplan and Nelson identified as the term that drives spontaneous baryogenesis. Dine and Thomas²⁷ and also Joyce, Turok and myself²³ have pointed out that there must be a mass squared suppression. Therefore this term is a pure gauge artefact. The proper (gauge covariant) form of f includes the kinetic momentum and a shifted ‘physical’ chemical potential

$$k_z = p_z \pm g_A Z, \quad \hat{\mu} = \mu \pm \gamma_w v_w g_A Z \quad (32)$$

where the signs are chosen appropriately. The source term

$$d_t \gamma_w(E + v_w k_z) = \gamma_w v_w \dot{k}_z = \gamma_w v_w (\partial_z E \pm g_A \dot{Z}) \quad (33)$$

vanishes in the zero mass limit as it should. I can now write down the linearized form of the *l.h.s.* of Eq. (28) as

$$d_t f = f'(\gamma_w(E + v_w p_z)) \left[\gamma_w v_w \dot{k}_z - d_t \mu - \gamma_w(E + v_w p_z) d_t \delta T - \gamma_w(p_z + v_w E) d_t v \right] \quad (34)$$

with $d_t \rightarrow \partial_{p_z} E \partial_z \simeq \frac{p_z}{E} \partial_z$ in the wall frame.

Rather than studying the full fluid equations I will simplify the problem by neglecting the temperature fluctuation. This is reasonably accurate when the scatterings that destroy δT are much more efficient than the decay processes that destroy μ . In the electroweak plasma this is satisfied to a good approximation. For a detailed treatment of the complete fluid equations see²⁸.

In order to obtain the fluid equations I integrate Eq. (34) in the wall frame over $\int d^3 p$ and $\int d^3 p \gamma_w(p_z + v_w E)$. I have chosen this Lorentz-like combination of E and

p_z since in this case the collision term has a simple form when evaluated in the fluid frame: only the velocity perturbation is damped and both chemical potential and temperature perturbations drop out. After a lengthy algebra I arrive at the fluid equations for particle minus antiparticle perturbations ($\mu = \mu(L) - \mu(\bar{L})$, $v = v(L) - v(\bar{L})$) in the rest frame of the wall

$$-a_{23}v_w\gamma_w\hat{\mu}' + \gamma_w\frac{1}{3}v' = -\Gamma_{\mu 1}[\hat{\mu}] - \Gamma_{\mu 1}^*[\hat{\mu} + 2\gamma_w v_w g_A Z] \quad (35)$$

$$a_{34}\gamma_w\hat{\mu}' - \gamma_w v_w v' + F_3 = -\Gamma_v v$$

$$F_3 = 3a_{14}v_w(g_A Z m^2)' \left(1 - \frac{v_w}{2} \ln \frac{1+v_w}{1-v_w}\right) \quad (36)$$

where $[\hat{\mu}]$ denotes the signed sum of chemical potentials for particles participating in the reaction, prime denotes ∂_z , $a_n = (n!\zeta_n/2\pi^2)[1 - 1/2^{n-1}]$ ($n > 1$), $a_1 = (\ln(2/m)/2\pi^2)[\ln m]$, $a_{23} = a_2/a_3 = (\zeta_2/3\zeta_3)[2/3]$, $a_{34} = a_3/a_4 = (\zeta_3/4\zeta_4)[6/7]$, $a_{14} = a_1/a_4 = (\ln 2/24)[8/7]$ and ζ is the Riemann ζ -function; to obtain the constants a_{ij} for bosons one should simply drop the factors in square brackets.

There are two sources of baryogenesis in these equations

- ▷ the classical force term $\propto (g_A Z m^2)'$ in the second equation and
- ▷ the hypercharge violating processes $\Gamma_{\mu 1}^*[2\gamma_w v_w g_A Z]$

The rates $\Gamma_{\mu 1}$ and Γ_v damp the chemical potential and velocity perturbations respectively. If one for example considers the top quark baryogenesis, the main tree level processes that contribute to $\Gamma_{\mu 1}$ are the helicity flipping top-gluon scattering with the emission of a charged higgs particle or a charged W boson and permutations of these. The process with the higgs emission (absorption) is faster due to its large Yukawa coupling. For quarks in addition there is also the anomalous strong sphaleron process that flips chirality. The hypercharge violating helicity flipping processes are $(m/\pi T)^2$ suppressed and therefore rather slow (see ²⁸). In this letter I shall focus on studying the effect of the classical force. In order to simplify the discussion I shall set all decay rates to zero. This is justified provided during the wall passage a small portion of the seeded perturbation decays *i.e.*

$$\frac{L}{v_w} \ll \Gamma_{\mu 1}, \Gamma_{\mu 1}^* \quad (37)$$

Also it is important that decays do not cut-off the diffusion tail in front of the wall. I will comment more on it below.

I now set all decay terms to zero $\Gamma_{\mu 1}, \Gamma_{\mu 1}^* \rightarrow 0$. Eq. (35) can be integrated to give

$$v = 3a_{23}v_w\hat{\mu} \quad (38)$$

since at $+\infty$ both $\hat{\mu} = v = 0$. Eq. (36) can then be written as

$$\bar{D}\hat{\mu}' + v_w\hat{\mu} = s' \equiv -\frac{F_3}{3a_{23}\Gamma_v}$$

$$\bar{D} = D \left(1 - 3v_w^2 \frac{a_{23}}{a_{34}}\right), \quad D = \frac{a_{34}}{3a_{23}\Gamma_v} \quad (39)$$

where D is the (plasma frame) diffusion constant. The inclusion of the temperature perturbations changes D only slightly; in that case $D = 1/3\Gamma_v$. We have calculated²⁸ the contribution of the quark-quark gluon exchange tree-level diagram and found $D \sim 6/T$, a similar contribution is expected from the quark-gluon scattering diagram, which would half the diffusion constant²⁹.

Note that close to the speed of sound $v_w \sim v_s = 1/\sqrt{3}$ in this approximation the fluid becomes *stiff*, i.e. transport is suppressed ($\bar{D}/v_w \ll L$) and there is only local baryogenesis:

$$\hat{\mu} = \frac{s'}{v_w} = -\frac{1}{3a_{23}v_w} \frac{F_3}{\Gamma_v} \quad (40)$$

which is suppressed as \bar{D}/Lv_w . This result should be taken with caution since the limit $\bar{D} \sim 0$ might signify the breakdown of the fluid approximation. In a more careful treatment one should include other types of perturbations, most notably δT . If one does so one finds that the perturbation at the source may be transported back to distances $\sim 1/\Gamma_v$. So generically there is no diffusion in front of the wall, but particles *do* diffuse behind the wall.

In order to write a general solution to Eq. (39) we write down the solution using the Greens function method

$$\partial_x G + \frac{v_w}{\bar{D}} G = \delta(x - y), \quad G = \theta(x - y) \exp\left[-\frac{v_w}{\bar{D}}(x - y)\right], \quad \hat{\mu} = \int G s' \quad (41)$$

for $\bar{D} > 0$. This form of the solution is quite useful since using partial integration it can be expanded in powers of $\bar{D}/v_w L$ for thick walls, or Lv_w/\bar{D} for thin walls. I leave this as an exercise. The Greens function in Eq. (41) is the response to a δ -function source and as such illustrates transport properties of the full solution Eq. (41): in this approximation when no particles decay all of the sourced perturbation gets transported into the diffusion tail in front. This tail extends to \bar{D}/v_w in front of the source which is large in the limit of a small velocity. A particle spends on average particle

$$\tau_{diff} \sim \frac{\bar{D}}{v_w^2} \quad (42)$$

diffusing in front of the wall. If this time is larger than any of the (zero mass) decay rates, then the diffusion tail will be suppressed as $\Gamma_\mu \bar{D}/v_w^2$ and accordingly the source for baryogenesis will be reduced.

In order to solve for the baryon number I may integrate Eq. (12) and find that it suffices to know the integral of $\hat{\mu}$ in front of the wall (for non-local baryogenesis) or on the wall (for local baryogenesis). Integrating Eq. (39) and assuming efficient transport such that $v_w L/\bar{D} \ll 1$, I find that *on the wall*,

$$\hat{\mu} = \frac{s}{\bar{D}} = -\frac{2 \ln 2}{3\zeta_3} \gamma_w v_w g_A Z m^2 V(v_w) \quad \gamma_w V(v_w) = \frac{1 - \frac{v_w}{2} \ln \frac{1+v_w}{1-v_w}}{1 - 3v_w^2 a_{23}/a_{34}} \quad (43)$$

In order to obtain the baryon number I as anticipated integrate Eq. (12) with all appropriate constants to find in the plasma frame

$$-\frac{1}{v_w \gamma_w} \int \dot{n}_B = n_B = \frac{3}{2} N_C \int \frac{\Gamma_{sph}}{V} (\mu_{t_L} - \mu_{\bar{t}_L}) \quad (44)$$

where $N_C = 3$ is the number of colors and Γ_{sph}/V is the sphaleron rate per unit volume. Note that I can easily obtain by integrating Eqs. (36) and (38) that $\int \hat{\mu} = \int_{wall} \hat{\mu} + \int_0^\infty \mu = 0$. This does not mean that baryon production vanishes because the sphaleron rate on the wall Eq. (8) is exponentially suppressed, indeed to a first approximation I can neglect the baryon production on the bubble wall to find that the net baryon to entropy ratio is

$$\frac{n_B}{s} = \frac{135 \ln 2}{2\pi^2 \zeta_3} \frac{\kappa_{sph} \alpha_w^4}{g_*} \int dz \frac{m^2 g_A Z}{T^2} V(v_w) \approx 4 \frac{\kappa_{sph} \alpha_w^4}{g_*} \int dz \frac{m^2 g_A Z}{T^2} V(v_w) \quad (45)$$

where $s = \frac{2\pi^2}{45} g_* T^3$ is the entropy density, with $g_* \approx 100$ the effective number of degrees of freedom, and we have restored the units of temperature. One should not forget that this result is applicable only in the limit of efficient transport, *i.e.* not too close to the speed of sound, when $\bar{D} \sim 0$ and only local baryogenesis is possible.

This result is remarkably simple - all dependence on the wall thickness and the diffusion constant drops out. The dependence on the wall velocity is rather weak; it is given by $V(v_w)$ of Eq. (43). For modest velocities $V(v_w) \sim (1 - v_w)^{3/2} / (1 - 3v_w^3 a_{23}/a_{34})$ so that baryon production becomes more efficient as the velocity increases; the growth stops when the condition for efficient transport gets violated: $\bar{D} \sim v_w L$. The asymmetry is quite large for the top quarks: $(m_t/T)^2 \sim 1/4$, so $n_B/s \sim 10^{-8} \kappa_{sph} \theta_{CP}$ where θ_{CP} characterizes the strength of CP violation. The m^2 dependence in Eq. (45) means that, at least with standard model-like Yukawa couplings, the top quark dominates the effect.

The calculation of the classical force effect above uses the opposite (WKB) approximation to those employed in quantum mechanical reflection calculations (thin walls^{25, 26}). The classical force calculation is in some respects ‘cleaner’, because the production of chiral charge and its diffusion are treated together. The classical force affects particles from all parts of the spectrum, mostly with typical energies $E \sim T$, and with no preferential direction, while the quantum mechanical effect comes mainly from particles with a very definite ingoing momentum perpendicular to the wall: $p_z \approx m_H$ (Higgs mass). The quantum result falls off exponentially with L as the *WKB* approximation becomes good. The quantum result also has a v_w^{-1} dependence coming from the diffusion time in the medium, which the classical result loses because the force term is proportional to v_w .

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8. References

1. A. Sakharov, *JETP Lett.* **5** (1967) 17; V. Kuzmin, *JETP Lett.* **12** (1970) 335.
2. A. M. Boesgaard and G. Steigman, *Ann. Rev. Astron. Astro.* **23** (1985) 319.
3. L. M. Krauss, preprint CWRU-P1-95, (1994), Talk given at Trends in Astroparticle Physics, Stockholm, Sweden, 22-25 Sep 1994; astro-ph/9501101; L. M. K. and P. J. Kernan, *Phys. Lett. B* **347** (1995) 347.
4. S. Dimopoulos and L. Susskind, *Phys. Lett.* **81 B** (1979) 416.
5. G. 't Hooft. *Phys. Rev. Lett.* **37** (1976) 8.
6. N. S. Manton, *Phys. Rev. D* **28** (1983) 2019; F.R. Klinkhamer and N. S. Manton, *Phys. Rev. D* **30** (1984) 2212.
7. V. Kuzmin, V. Rubakov and M. Shaposhnikov, *Phys. Lett.* **155B** (1985) 36.
8. P. Arnold and L. McLerran, *Phys. Rev. D* **36** (1987) 581; **D37** (1988) 1020.
9. M. E. Shaposhnikov, *JETP Lett.* **44** (1986) 465; *Nucl. Phys. B* **287** (1987) 757; *Nucl. Phys. B* **299** (1988) 797.
10. Guy Moore has recently shown that the time during which sphalerons are active should be $H^{-1}d(E_{sph}/t)/dT \sim 1000H^{-1}$, rather than H^{-1} ; private communication.
11. D. Yu. Gregoriev, V. A. Rubakov and M. E. Shaposhnikov, *Phys. Lett.* **216 B** (1989) 172; J. Ambjorn, T. Askgaard, H. Porter and M. Shaposhnikov, *Phys. Lett.* **244B** (1990) 497; *Nucl. Phys. B* **353** (1991) 346.
12. P. Arnold and O. Espinosa, *Phys. Rev. D* **47** (1993) 3546; W. Buchmüller, Z. Fodor, and A. Hebecker, *Phys. Lett. B* **331** (1994) 131; W. Buchmüller, Z. Fodor, T. Helbig and D. Walliser, *Ann. Phys.* **234** (1994) 260, hep-ph/9303251.
13. K. Farakos, K. Kajantie, K. Rummukainen, and M. Shaposhnikov, preprint CERN-TH-7220-94 (1994); hep-lat/9412091
14. A private communication with M. Shaposhnikov.
15. G. Farrar and M. Shaposhnikov, *Phys. Rev. Lett.* **70** (1993) 2833; *Phys. Rev. D* **50** (1994) 774.
16. M. B. Gavela, P. Hernandez, J. Orloff and O. Pene, *Mod. Phys. Lett.* **9A** (1994) 795; M. B. Gavela, M. Losano, J. Orloff and O. Pene, *Nucl. Phys. B* **430** (1994) 345 and 382, hep-ph/9406288 and 9406289.
17. P. Huet and E. Sather, *Phys. Rev. D* **51** (1995) 379, hep-ph/9404302.
18. S. Nasser and N. Turok, Princeton preprint PUPT-1456 (1994), hep-ph/9406270.
19. A. Cohen and D. Kaplan, *Phys. Lett.* **199 B** (1987) 251.
20. M. Dine, P. Huet, R. Singleton and L. Susskind, *Phys. Lett. B* **257** (1991) 351.
21. N. Turok, J. Zadrozny, *Phys. Rev. Lett.* **65** (1990) 2331; *Nucl. Phys. B* **358** (1991) 471; L. McLerran, M. E. Shaposhnikov, N. Turok and M. Voloshin, *Phys. Lett.* **256B** (1991) 451.
22. A. Cohen, D. Kaplan, and A. Nelson, *Phys. Lett. B* **263** (1991) 86.

- 23. M. Joyce, T. Prokopec and N. Turok, *Phys. Lett. B* **338** (1994) 269, hep-ph/9401351.
- 24. A. Cohen, D. Kaplan and A. Nelson, *Phys. Lett. B* **245** (1990) 561; *Nuc. Phys. B* **349** (1991) 727..
- 25. A. Cohen, D. Kaplan and A. Nelson, *Nuc. Phys. B* **373** (1992) 453.
- 26. M. Joyce, T. Prokopec and N. Turok, Princeton preprint PUPT-1437, *Phys. Lett. B*, in press (1995).
- 27. M. Dine and S. Thomas, *Phys. Lett. B* **328** (1994) 73, hep-ph/9401265.
- 28. M. Joyce, T. Prokopec and N. Turok, Princeton preprint PUPT-1496 (1994).
- 29. G. Moore, T. Prokopec, in preparation.